Grade 5 Module 5: Addition and Multiplication with Volume and Area

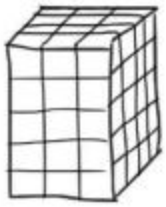
New or Recently Introduced Terms

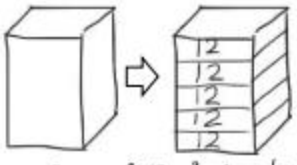
- Base (one face of a three-dimensional solid—often thought of as the surface upon which the solid rests)
- Bisect (divide into two equal parts)
- Cubic units (cubes of the same size used for measuring)
- Height (adjacent layers of the base that form a rectangular prism)
- Unit cube (cube whose sides all measure 1 unit; cubes of the same size used for measuring volume)
- Volume of a solid (measurement of space or capacity)
- Angle (the union of two different rays sharing a common vertex)
- Area (the number of square units that covers a two-dimensional shape)
- Cube (three-dimensional figure with six square sides)
- Degree measure of an angle (subdivide the length around a circle into 360 arcs of equal length; a central angle for any of these arcs is called a *one-degree angle* and is said to have angle measure 1 degree)
- Face (any flat surface of a three-dimensional figure)
- Kite (quadrilateral with two equal sides that are also adjacent; a kite can be a rhombus if all sides are equal)
- Parallel lines (two lines in a plane that do not intersect)
- Parallelogram (four-sided closed figure with opposite sides that are parallel)
- Perpendicular (two lines are *perpendicular* if they intersect, and any of the angles formed between the lines are 90˚ angles)
- Perpendicular bisector (line that cuts a line segment into two equal parts at 90˚)
- Plane (flat surface that extends infinitely in all directions)
- Polygon (closed figure made up of line segments)
- Quadrilateral (closed figure with four sides)
- Rectangle (quadrilateral with four 90˚ angles)
- Rectangular prism (three-dimensional figure with six rectangular sides)
- Rhombus (parallelogram with equal sides)
- Right angle (angle formed by perpendicular lines; angle measuring 90˚)
- Right rectangular prism (rectangular prism with only 90˚ angles)
- Solid figure (three-dimensional figure)
- Square units (squares of the same size, used for measuring)
- Three-dimensional figures (solid figures)
- Trapezoid (quadrilateral with at least one pair of parallel sides)
- Two-dimensional figures (figures on a plane)

Topic A: Concepts of Volume

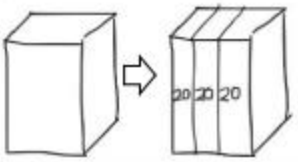
In Topic A, students extend their spatial structuring to three dimensions through an exploration of volume. They come to see volume as an attribute of solid figures and understand that cubic units are used to measure it. Using unit cubes, both customary and metric, students build three-dimensional shapes, including right rectangular prisms, and count to find the volume. By developing a systematic approach to counting the unit cubes, they make connections between area and volume.

Next, students pack rectangular prisms made from nets with centimeter cubes. This helps them to visualize the layers of cubic units that compose volumes, an understanding that allows them to reasonably predict the number of cubes required to fill the containers and then test their predictions by packing the containers. Finally, students compose and decompose a rectangular prism from and into layers of unit cubes, and reason that the number of unit cubes in a single layer corresponds to the number of unit squares on a face. They begin to conceptualize the layers themselves, oriented in any one of three directions, as iterated units.

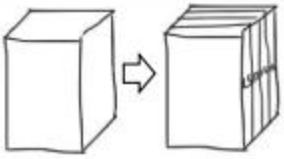




12 cm³ + 12 cm³ + 12 cm³ + 12 cm³ + 12 cm³ = 60 cubic cm
5 x 12 cubic cm = 60 cubic cm



20 cm³ + 20 cm³ + 20 cm³ = 60 cubic cm
3 x 20 cubic cm = 60 cubic cm

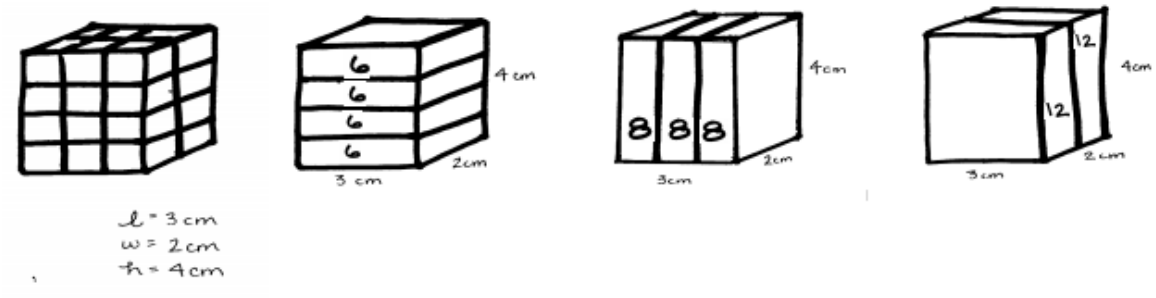


15 cm³ + 15 cm³ + 15 cm³ + 15 cm³ = 60 cubic cm
4 x 15 cubic cm = 60 cubic cm

Number of Layers	Cubes in Each Layer	Volume
5	4	20 cm^3
2	10	20 cm^3
2	10	20 cm^3

Topic B: Volume and the Operations of Multiplication and Addition

Concrete understanding of volume and multiplicative reasoning come together in Topic B as the systematic counting from Topic A leads naturally to formulas for finding the volume of a right rectangular prism. Students come to see that multiplying the edge lengths or multiplying the height by the area of the base yields an equivalent volume to that found by packing and counting unit cubes.



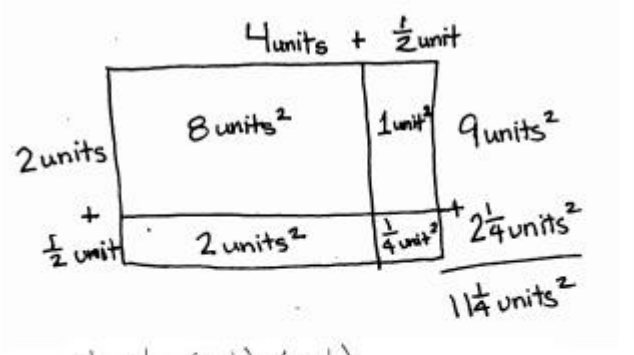
Cubes in Each Layer	Number of Layers	Volume
(3 × 2)	4	$24cm^3$
(2 × 4)	3	$24\ cm^3$
(3 × 4)	2	$24\ cm^3$

Next, students solidify the connection between volume as *packing* with volume as *filling* by comparing the amount of liquid that fills a container to the number of cubes that can be packed into it. This connection is formalized as students see that 1 cubic centimeter is equal to 1 milliliter. Complexity increases as students use their knowledge that volume is additive to partition and calculate the total volume of solid figures composed of non-overlapping rectangular prisms.

Word problems involving the volume of rectangular prisms with whole number edge lengths solidify understanding and give students opportunity to reason about scaling in the context of volume. This topic concludes with a design project that allows students to apply the concepts and formulas they have learned throughout Topics A and B to create a sculpture of a specified volume composed of varied rectangular prisms with parameters stipulated in the project description.

Topic C: Area of Rectangular Figures with Fractional Side Length

In Topic C, students extend their understanding of area as they use rulers and set squares to construct and measure rectangles with fractional side lengths and find their areas. They apply their extensive knowledge of fraction multiplication to interpret areas of rectangles with fractional side lengths and solve real world problems involving these figures, including reasoning about scaling through contexts in which volumes are compared. Visual models and equations are used to represent the problems through the Read-Draw-Write protocol.

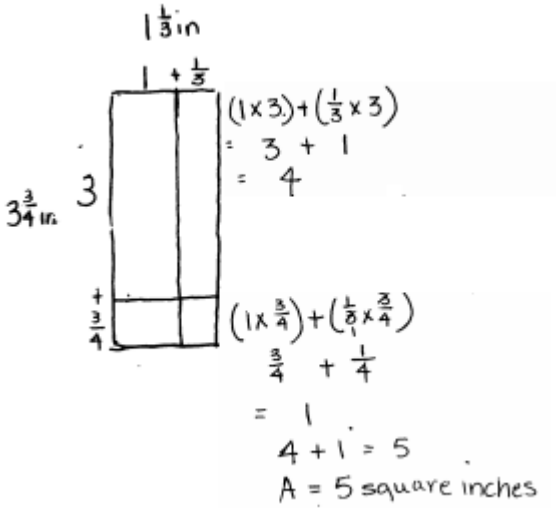


$4\frac{1}{2} \times 2\frac{1}{2} = (4+\frac{1}{2}) \times (2+\frac{1}{2})$

$= (2\times4) + (2\times\frac{1}{2}) + (\frac{1}{2}\times4) + (\frac{1}{2}\times\frac{1}{2})$

$= 8 + 1 + 2 + \frac{1}{4}$

$= 11\frac{1}{4}$



Topic D: Drawing, Analysis, and Classification of Two-Dimensional Shapes

In Topic D, students draw two-dimensional shapes in order to analyze their attributes, and then use those attributes to classify them. Familiar figures, such as parallelograms, rhombuses, squares, and trapezoids, have all been defined in earlier grades, and by Grade 4, students have gained an understanding of shapes beyond the intuitive level. Grade 5 extends this understanding through an in-depth analysis of the properties and defining attributes of quadrilaterals.

Grade 4's work with the protractor is applied in this topic in order to construct various quadrilaterals. Using measurement tools illuminates the attributes used to define and recognize each quadrilateral. Students see, for example, that the same process that they used to construct a parallelogram will also produce a rectangle when all angles are constructed to measure 90°. Students then analyze defining attributes and create a hierarchical classification of quadrilaterals.

A trapezoid:

- Is a quadrilateral in which at least one pair of opposite sides is parallel.

A parallelogram:

- Is a quadrilateral in which both pairs of opposite sides are parallel.

A rhombus:

- Is a quadrilateral with all sides of equal length.

A rectangle:

- Is a quadrilateral with four right angles.

Formal Definition of a Quadrilateral:
(Only the first bullet is introduced today.)

- Consists of four different points A, B, C, D in the plane and four segments, $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$,
- Is arranged so that the segments intersect only at their endpoints, and
- Has no two adjacent segments that are collinear.

A square:

- Is a rhombus with four right angles.
- Is a rectangle with four equal sides.

A kite:

- Is a quadrilateral in which two consecutive sides have equal length, and
- Has two remaining sides of equal length.